Are Corporate Bond Market Returns Predictable?

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ABSTRACT

This paper examines predictability of corporate bond returns using the transaction-based index data for the period from October 1, 2002 to April 30, 2009. We find evidence of significant serial and cross-serial dependence in daily investment-grade and high-yield bond returns. The serial dependence exhibits a complex nonlinear structure. Stock market returns lead bond market returns and this lead-lag relation is stronger between stocks and high-yield straight bonds. These findings are robust to various model specifications and provide important implications for modeling term structure of defaultable bonds.

JEL classification: G14; G12

Keywords: return predictability; generalized spectrum; autocorrelation; causality; nonlinearity; bond pricing; market efficiency.
1. Introduction

One of the most enduring issues in finance and economics is the question of whether returns on risky assets are predictable. This important issue has been the focus of an extensive literature on asset prices dated back more than a century. Despite an enormous amount of past efforts, whether future asset price changes can be meaningfully predicted is still a subject of on-going debates and intensive empirical research (see, for example, Cochrane and Piazzesi, 2005; Ang and Bekaert, 2007; Cochrane, 2008; Campbell and Thompson, 2008; Spiegel, 2008; Welch and Goyal, 2008; Pastor and Stambaugh, 2009; Rapach, Strauss and Zhou, 2010).

The literature of asset return predictability has focused on the stock market. There is substantial evidence that stock returns are predictable either by past price changes or economic variables (see, among many others, Lo and MacKinlay, 1988, 1990; Fama and French, 1988a, 1988b; Pesaran and Timmermann, 1995; Lanne, 2002; Lewellen, 2004; Campbell and Yogo, 2006; Ang and Bekaert, 2007; Cochrane, 2008; Pastor and Stambaugh, 2009; Erik, 2010; Rapach et al., 2010). Recent efforts have been directed to identifying the predictive components of asset returns at different return horizons, evaluating predictive power of predictors using more robust tests, and determining how much predictability is compatible with efficiency consistent with risk-based asset pricing models.

Notwithstanding extensive research on equity return predictability, there are only a few studies on corporate bond return predictability (see Keim and Stambaugh, 1986; Kwan, 1996; Hotchkiss and Ronen, 2002; Downing, Underwood and Xing, 2009) and empirical evidence is inconclusive. Kwan (1996) shows that significant negative contemporaneous correlation exists between returns of individual stocks and yield changes of bonds issued by the same firm, and that stock returns predict future bond yield changes. Unlike Kwan (1996), Hotchkiss and Ronen (2002) find that corporate bond returns cannot be predicted by past stock returns based on a sample of 55 high-yield bonds. By contrast, Downing, Underwood and Xing (2009) show that stock returns predict convertible bond returns in all rating categories but predict returns of only BBB-

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1 Cornell and Green (1991) and Blume, Keim and Patel (1991) examine the sensitivity of high-yield bond returns to interest rates.
and junk-rated nonconvertible bonds.

In this paper, we examine predictability of corporate bond returns in a narrow sense by focusing on serial dependence and causality tests. Similar to mainstream equity premium studies, we examine return predictability at the aggregate level. We employ bond market index data constructed from transaction prices, instead of dealer quotes used in many previous studies (see, for example, Kwan, 1996; Gebhardt et al. 2005). Our empirical analysis draws heavily on the rich literature in random walk and causality tests (Campbell, Lo and MacKinlay, 1997; Granger, 1969; Engle and Kroner, 1995; Engle, 2002). Similar to Chen and Maringer (2011), we account for nonlinearity in corporate bond index returns. Standard methods of return predictability tests are not robust to nonlinear dependence. To overcome this problem, we employ an advanced generalized spectral method (Hong, 1999; Hong and Lee, 2005, 2007) to detect nonlinear dependencies in returns and to perform robust tests. Furthermore, we conduct causality tests on bond and stock returns by accounting for heteroskedasticity in the error term and potential nonlinearity in the causal relationship.

Knowledge of bond price dynamics is important for formulating optimal strategies for asset allocation and hedging. Corporate bonds account for a significant portion of investors’ wealth with a market size near 6 trillion (see Abhyankar and Gonzalez, 2009). So understanding the corporate bond price dynamics is essential for academics and practitioners (Chan and Wu, 1995; Chen and Maringer, 2011). This paper, to the best of our knowledge, is the first paper that provides comprehensive time-series analysis on serial and cross-serial dependencies in transaction-based corporate bond index returns.

We find strong evidence of serial and cross-serial dependence in corporate bond market returns. Empirical analysis reveals a complicated nonlinear structure of serial dependence in bond returns. Both investment-grade and high-yield bond returns can be predicted by past stock market returns and the predictive relation is much stronger between stocks and high-yield bonds. By contrast, there is little evidence that stock returns can be predicted by past bond returns. These findings persist even after accounting for the effects of conditional heteroskedasticity, volatility-induced mean return changes,

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2 See also a similar remark by Blume, Keim and Patel (1991). Their study focuses on returns and risks of low-grade bonds.
and time-varying interest rates.

The remainder of the paper is organized as follows. In Section 2, we describe the empirical methodology for testing linear and nonlinear serial dependence in returns. In Section 3, we propose vector autoregressive regression models (VAR) and Granger-causality tests with homoskedastic and heteroskedastic returns. In Section 4, we present empirical test results for serial and cross-serial dependence in stock and bond market returns, and examine the robustness of results to model specifications and return measures. In Section 5, we examine the sensitivity of corporate bond returns to concurrent and lagged stock and government bond returns. Finally, we summarize our findings and conclude the paper in Section 6.

2. Tests of Serial Dependence in Returns

A fundamental issue in asset pricing is whether asset returns can be predicted using historical information.\(^3\) In this section, we propose tests on predictive models with past returns. We first set forth hypotheses on serial dependence in conditional means of bond returns and then discuss tests on serial correlation and the spectral test on the martingale difference sequence (MDS) in returns.

2.1 Test hypothesis

Let \( \{X_t\} \) be a weakly stationary return process with \( E(X_t) = \mu \). The hypotheses of interest are

\[ H_0: \ E(X_t|I_{t-1}) = \mu \]

against

\[ H_A: \ E(X_t|I_{t-1}) \neq \mu \]

The test above deals primarily with the question of whether there exists a dependence structure in conditional mean. It does not impose any assumption on higher order moments. To the extent that the conditional variance \( h_t = \text{var}(X_t|I_{t-1}) \) or other higher-order conditional moments are time-varying, higher-moment properties could affect the test statistic for \( H_0 \). On the other hand, as no model parameter estimation is involved here, there is no need to consider the potential impact of uncertainty in parameter estimation on the test statistic. The information set \( I_{t-1} \) in the conditional mean test may contain only the

\(^3\) See Campbell, Lo and MacKinlay (1997).
past history of \(X_t\) or the past history of both \(X_t\) and other variables. When the information set contains only the history of the own variable, \(I_{t-1} = \{X_{t-1}, X_{t-2}, \ldots\}\), it is a test of serial dependence in conditional mean. By contrast, when the information set includes the history of another variable, \(I_{t-1} = \{X_{t-1}, Y_{t-1}, \ldots\}\), the test involves cross dependence in conditional mean.

Given that the information set contains only the own history, \(I_{t-1} = \{X_{t-1}, X_{t-2}, \ldots\}\), under the null hypothesis \(H_0\) of \(E(X_t - \mu | I_{t-1}) = 0\), the martingale difference sequence (after demeaning) implies that

(i) \(\{X_t\}\) is serially uncorrelated or white noise (WN),

\[
\gamma(j) = \text{cov}(X_t, X_{t-j}) = 0, \text{ for all } j > 0
\]

or equivalently,

(ii) \(\{X_t\}\) has a flat spectrum

\[
h(\omega) = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \text{cov}(\varepsilon_t, \varepsilon_{t-j}) e^{-ij\omega} = \frac{1}{2\pi} \gamma(0) \text{ for all } \omega \in [-\pi, \pi].
\]

Thus, we can test \(H_0\) by investigating whether \(\gamma(j) = 0\) for all \(j > 0\) or alternatively, we can examine whether \(\{X_t\}\) possesses a flat spectrum.

The hypothesis can be tested using standard autocorrelation tests such as Box-Pierce-Ljung tests (Box and Pierce, 1970; and Ljung and Box, 1978). Additional tests include the spectral distribution test (Durlauf, 1991), variance ratio tests of Cochrane (1988) and Lo and MacKinlay (1988), and the spectral density test of Hong (1996). In empirical tests, there is a subtle difference between a white noise and an MDS. Although an MDS is a white noise, a white noise may not be an MDS. This can be illustrated by a simple example involving a nonlinear moving average process

\[
X_t = \alpha \varepsilon_{t-1} + \varepsilon_t + \varepsilon_t, \text{ where } \varepsilon_t \sim i.i.d.(0, \sigma^2).
\]

For this process, \(\text{cov}(X_t, X_{t-j}) = 0\) for all \(j > 0\), but \(E(X_t | I_{t-1}) = \alpha \varepsilon_{t-1} \neq 0\). Thus, this process is a white noise but not an MDS. A problem for the tests based on the autocovariance function \(\gamma(j)\) or the power spectrum is that they cannot detect such non-MDS alternatives that exhibit zero autocorrelation.

Economic behavior and the investor’s attitude toward risk and returns can be quite complicated. As these factors affect returns, the return process may not be characterized by a linear function. Several methods have been proposed to test if a return series is a
martingale difference sequence. These include development of the indicator function test (Domínguez and Lobato, 2003; and Park and Whang, 2004) and the generalized spectral test (Hong, 1999; Hong and Lee, 2003; and Hong and Lee, 2005, 2007). In the following, we discuss different methods for serial dependence tests.

2.2 Autocorrelation tests

Define the sample autocovariance function at lag $j$ as

$$\hat{\gamma}(j) = T^{-1} \sum_{t=j+1}^{T} (X_t - \bar{X})(X_{t-j} - \bar{X}), \quad j = 0, \pm 1, \ldots, \pm (T-1)$$

where $\bar{X}$ is the sample mean. The sample autocorrelation functions are

$$\hat{\rho}(j) = \hat{\gamma}(j)/\hat{\gamma}(0), \quad j = 0, \pm 1, \ldots, \pm (T-1).$$

Under the i.i.d. assumption on $\{X_t\}$, one can employ the Box and Pierce (1970) test to detect autocorrelation:

$$BP(p) = T \sum_{j=1}^{p} \hat{\rho}^2(j) = \sum_{j=1}^{p} \left[ \sqrt{T} \hat{\rho}(j) \right]^2 \rightarrow \chi^2_p.$$  

To improve the size of the Box-Pierce test in finite samples, Ljung and Box (1978) suggest a modified test

$$LB(p) = T(T+2) \sum_{j=1}^{p} (T-j)^{-1} \hat{\rho}^2(j) \rightarrow \chi^2_p.$$  

The asymptotic distribution of LB statistics holds with conditional homoskedasticity $\text{var}(X_t|I_{t-1}) = \sigma^2$. The LB test lacks power in the presence of conditional heteroskedasticity, which is unfortunately quite common in financial markets. Nevertheless, the LB test remains to be a useful diagnostic tool because it is conceptually intuitive and relatively straightforward to apply to financial data.

2.3 Variance ratio tests

Variance ratio tests have been used widely in random walk tests of stock returns. The basic principle behind the variance ratio test is that variance of the increments of a random walk is linear in the sampling interval. For example, if asset prices follow a random walk process, the variance of monthly returns will be four times as large as the variance of weekly returns. Let $\sum_{j=1}^{p} X_{r-j}$ be the cumulative return over a period of $p$ days. Then, under the null hypothesis $H_0$,
\[
\frac{\text{var}(\sum_{j=1}^{p} X_{t-j})}{p \text{ var}(X_t)} = \frac{p \gamma(0) + 2p \sum_{j=1}^{p} (1 - j/p) \gamma(j)}{p \gamma(0)} = 1,
\]

if the autocovariance functions \( \gamma(j) = 0 \) for all \( j \neq 0 \). This unique property of the variance ratio can be utilized to test \( H_0 \) since any departure from unity is evidence against it.

Under conditional homoskedasticity, Lo and MacKinlay (1988) show that the variance ratio has the following limiting distribution:

\[
VR(p) = \frac{p \gamma(0) + 2p \sum_{j=1}^{p} (1 - j/p) \hat{\gamma}(j)}{p \hat{\gamma}(0)} \to^d N\left[0, 2(2p-1)(p-1)/3p\right].
\]

However, volatilities of returns in financial markets often change over time. Under the null \( H_0 \) with conditional heteroskedasticity, the variance ratio has the following asymptotical distribution instead,

\[
VR(p) = \sqrt{T/p} \sum_{j=1}^{p} (1 - j/p) \hat{\gamma}(j)/\sqrt{\hat{\gamma}_2(j)} \to^d N\left[0, 2(2p-1)(p-1)/3p\right]
\]

where \( \hat{\gamma}(j) \) is the sample autocovariance function and

\[
\hat{\gamma}_2(j) = T^{-1} \sum_{t=|j|+1}^{T} (X_t - \bar{X})^2 (X_{t-j} - \bar{X})^2, \quad j = 0, \pm1, \ldots, \pm(T-1).
\]

When \( p \to \infty \) as \( T \to \infty \), the variance ratio test statistic is asymptotically equivalent to

\[
VR(p) = \sqrt{T/p} \sum_{j=1}^{p} (1 - j/p) \hat{\rho}(j) = \frac{\pi}{2} \sqrt{T/p} \left[ \hat{f}(0) - \frac{1}{2\pi} \right]
\]

where \( \hat{f}(0) \) is a kernel-based normalized spectral density estimator at frequency zero with the Bartlett kernel \( K(z) = (1 - |z|) 1(|z| \leq 1) \), and \( 1(|z| \leq 1) \) is an indicator function that equals one if the random variable \( z \) is between \([-1,1]\) and zero, otherwise.

### 2.4 The generalized spectral test

Hong (1999), Hong and Lee (2003), and Hong and Lee (2005, 2007) propose a more powerful approach to test the MDS hypothesis based on the generalized spectral density. The generalized spectral density of \( \{X_t\} \) can be written as

\[
f(\omega, u, v) = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \sigma_j(u, v) e^{-j\omega}, \quad \omega \in [-\pi, \pi], -\infty < u, v < \infty
\]
where

\[ \sigma_j(u, v) = \text{cov} \left( e^{iuX_t}, e^{ivX_{t-j}} \right) = \phi_j(u, v) - \phi_j(u) \phi_j(v) \]

is the generalized covariance function, and \( \phi_j(u, v) = E \left( e^{iuX_t} e^{ivX_{t-j}} \right) \) and \( \phi_j(u) = E \left( e^{iuX_t} \right) \) are the pairwise joint characteristic function at lag \( j \) and the marginal characteristic function of \{X_t\}, respectively.

The generalized spectral test has several advantages. First, one can use this method to test whether there exists serial dependence in mean or not where the dimension of the information set can be infinite. Second, the generalized spectral test method can be used to check dependence in conditional mean \( E(X_t|X_{t-j}) = E(X_t) \) for all \( j > 0 \), and so it is a more general test on the MDS than either the autocorrelation or variance ratio test. Third, since the first-order derivative of the generalized spectral density function is always flat no matter whether higher-order conditional moments are changing over time or not, one can construct a test for the MDS that is robust to time-varying higher-order moments of any unknown form based on this unique property. Lastly, the method can be used to check a large number of lags and it naturally discounts higher order lags which is consistent with the stylized fact that the economic behavior is more strongly affected by recent events than remote past events.4

Hong and Lee (2005, 2007) develop a test statistic to check whether \( \varepsilon_t = X_t - g(I_{t-1}, \theta) \) satisfies \( E(\varepsilon_t|I_{t-1}) = 0 \) where \( g(I_{t-1}, \theta) = \mu \). The test statistic is

\[ M(p) = \left[ \sum_{j=1}^{T-1} k^2(j/p)(T-j) \left| \hat{\phi}_j(0, 0) \right|^2 dW(v) - \hat{C}_j(p) \right] / \sqrt{\hat{D}_j(p)} \]

where

\[ \hat{C}_j(p) = \hat{S}^2 \left[ \frac{1}{T} \sum_{r=1}^{T} \hat{h}_r(v) \right]^2 dW(v) \sum_{j=1}^{T-1} k^2(j/p), \]

\[ \hat{D}_j(p) = 2\hat{S}^2 \left[ \int \left[ \frac{1}{T} \sum_{r=1}^{T} \hat{h}_r(u) \hat{h}_r(v) \right]^2 dW(u)dW(v) \sum_{j=1}^{T-1} k^4(j/p), \]

and \( \hat{S}^2 = \frac{1}{T} \sum_{j=1}^{T} \hat{\varepsilon}_j^2 \). \( W(\cdot): R \rightarrow R^+ \) is a nondecreasing function that sets weights symmetrically around zero. An example of \( W(\cdot) \) is the \( N(0,1) \) cumulative density function. The

4 For the unique advantages and details of the properties of this test method, see Hong (1996, 1999) and Hong and Lee (2005).
description for the variables in $M(p)$ and development of the test statistic is in the Appendix. Hong and Lee (2005) show that $M(p) \overset{d}{\rightarrow} N(0,1)$ under $H_0$; otherwise, $M(p)$ goes to positive infinity as the sample size increases. The MDS hypothesis is rejected if $M(p)$ is greater than the critical value of $N(0,1)$ at a given significance level.

3. Models for Causality Tests

Both bonds and stocks are different contingent claims issued by the same firm on the cash flow of the same underlying asset. Information about the expected value of the firm’s asset should therefore affect prices of bonds and stocks in the same direction. For example, favorable earnings news increases firm value and stock price. According to the structural model, expected default probability depends on the firm asset value. As firm value appreciates, default risk decreases and bond price increases. If stock and bond markets are equally efficient, this will induce a positive contemporaneous correlation between corporate bond and stock returns. On the other hand, volatility affects bond and stock prices in an opposite direction. A stock can be viewed as a call option on the firm’s underlying asset value and a bond can be viewed as a portfolio long in default-free asset and short in a put option on the firm value (see Merton, 1974). Information that increases asset return volatility but not the mean value of the firm should increase stock price and decrease bond price, resulting in a negative contemporaneous correlation between changes in stock and bond prices. Thus, the direction of the contemporaneous correlation between stock and bond returns depends on the nature of the information signal.

Conversely, if stock and bond markets are not equally efficient, due to either market frictions or other reasons, one market will impound information into prices faster than the other. If the stock price responds to new information faster than the bond price, stock returns will lead bond returns.\(^5\)

In this section, we propose tests on whether the past return in one market can predict the return in another market. Conventional tests on the lead-lag relationship rely on the VAR model and the Granger causality method. Aside from the standard linear causality test, we account for conditional heteroskedasticity in the error term using the GARCH

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\(^5\) Previous studies have shown that the speed of price adjustment to information depends on the degree of investor heterogeneity, and order flows in the market (see Green, 2004; and Brandt and Kavajecz, 2004).
model and the effect of volatility on the conditional mean return using the GARCH-M model. Considering the nonlinear structure in the return generating process produces more robust tests for the dynamic relationship between the two markets.

Let $I_{t-1}^k = \{I_{t-1}^{Z_1}, I_{t-1}^{Z_2}, \ldots, I_{t-1}^{Z_k}\}$, where $I_{t-1}^{Z_1} = \{Z_{t-1}, Z_{t-2}, \ldots\}$ to $I_{t-1}^{Z_k} = \{Z_{k,t-1}, Z_{k,t-2}, \ldots\}$ represent the historical information sets for $k$ variables. The variable $\{Z_t\}$ is Granger-caused by $\{Z_{2t}, Z_{3t}, \ldots, Z_{kt}\}$ with respect to the information set $I_{t-1}^2$ if $E(Z_{2t} | I_{t-1}^2) \neq E(Z_{2t} | I_{t-1}^1)$. The dynamic relation between variables can be conveniently cast in a vector autoregressive model

$$Z_t = \delta + \Phi_1 Z_{t-1} + \ldots + \Phi_p Z_{t-p} + \varepsilon_t$$

where $Z_t = (Z_{t1}, \ldots, Z_{tk})^T$, $t = 1, 2, \ldots, N$, is a $k$-dimensional vector of variables of interest, $p$ is the lag order, $\varepsilon_t = (\varepsilon_{t1}, \ldots, \varepsilon_{tk})^T$ is a vector of error terms with $E(\varepsilon_t) = 0$, $E(\varepsilon_t \varepsilon_t^T) = \Sigma$ and $E(\varepsilon_s \varepsilon_t^T) = 0$ for $s \neq t$, $\delta = (\delta_1, \ldots, \delta_k)^T$ is a constant vector, and $\Phi_j$, $j = 1, \ldots, p$ is a $k \times k$ matrix of response coefficients. This VAR system serves as the basic framework for causality tests.

### 3.1 Linear Granger causality test

Consider the bivariate case ($k=2$) where $Z_t = (Z_{t1}, Z_{t2})^T$ are two stationary time series. Standard linear causality tests can be performed using the bivariate VAR system:

$$Z_{t1} = a_0 + \sum_{j=1}^{p} a_j Z_{t1-j} + \sum_{j=1}^{p} b_j Z_{t2-j} + \varepsilon_t,$$

$$Z_{t2} = c_0 + \sum_{j=1}^{p} c_j Z_{t1-j} + \sum_{j=1}^{p} d_j Z_{t2-j} + \nu_t.$$

$Z_{t1}$ is Granger-caused by $Z_{t2}$ if some $b_j$ are not zero. We can test whether $b_j = 0$ jointly using the $F$ test. This test assumes conditional homoskedasticity, or $\text{var}(\varepsilon_t | I_{t-1}) = \sigma^2$, asymptotically.

### 3.2 Nonlinear Granger causality test

Standard Granger causality tests are not robust to heteroskedasticity. To overcome this problem, we modify the above model to accommodate conditional heteroskedasticity in
asset returns:

\[ Z_t = \delta + \Phi_1 Z_{t-1} + \ldots + \Phi_p Z_{t-p} + \varepsilon_t \]

where \( \varepsilon_t \mid I_{t-1} \sim N(0, H_t) \), and \( H_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix} \) is the conditional variance-covariance matrix. \( H_t \) can have a dynamic conditional correlation (DCC) representation (Engle, 2002), a BEKK representation (Baba, Engle, Kraft, Kroner, 1990; Engle and Kroner, 1995), or a BEW representation (Bollerslev, Engle and Wooldridge, 1988). In empirical investigation, we adopt the BEKK representation in bivariate GARCH (1,1), that is,

\[ H_t = c^T c + a^T \varepsilon_{t-1} \varepsilon_{t-1}^T a + g^T H_{t-1} g, \]

where

\[ c = \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix}, \quad a = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad g = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}, \quad \varepsilon_t = \begin{bmatrix} \varepsilon_{1,t-1} \\ \varepsilon_{2,t-1} \end{bmatrix}. \]

### 3.3 The GARCH-M model

Both linear and nonlinear Granger causality tests deal with the cross dependence in returns without considering the effects of higher-order moments on the mean return. It is possible that some higher-order moments of returns may affect the mean return of an asset. A classic example is the GARCH-M effect where the second moment (conditional variance) affects conditional mean returns (see Hamao, Masulis and Ng, 1990).

Considering the GARCH-M effect in a bivariate VAR-GARCH (1,1) setting, we have

\[ Z_t = \delta + \Psi_1 Z_{t-1} + \ldots + \Phi_p Z_{t-p} + \Psi_1 \widetilde{H}_t + \varepsilon_t, \quad Z_t = [Z_{1,t}, Z_{2,t}]^T \]

where \( \Psi_1 = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} \\ \Psi_{21} & \Psi_{22} & \Psi_{23} \end{bmatrix} \), \( \widetilde{H}_t = [h_{11,t}, h_{12,t}, h_{22,t}]^T \), and \( \varepsilon_t \mid I_{t-1} \sim N(0, H_t) \). The conditional variance-covariance matrix \( H_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix} \) again can be formulated by a BEKK representation. We next turn to empirical estimation.

### 4. Data and Empirical Results

#### 4.1 Data

Data include daily returns of the high-yield corporate bond index \( r_{nbbh} \), the
investment-grade corporate bond index \((r_{nbbi})\), the S&P 500 stock index \((r_{sp500})\) and the S&P 500 index futures \((r_{futures})\). Corporate bond return data are based on NASD-Bloomberg US High-Yield and Investment-Grade Bond Indices, which are downloaded from the Bloomberg system. These indices are constructed from actual transaction prices of the active fixed-coupon bonds represented by the TRACE system of the NASD that disseminates OTC trades for all publicly traded corporate bonds. The index price is volume-weighted average price generated from TRACE transactions.\(^6\) The index basket excludes zero-coupon and convertible bonds and bonds set to mature before the last day of the month for which index rebalance occurs. The sample period of the data is from October 1, 2002 to April 30, 2009.

Using the corporate bond index data has several advantages. First, the index is broad-based, well representing the whole corporate bond market. Second, the index is generated from transaction prices, instead of dealers’ quotes or matrix prices which are not representative of actual transactions (see Gebhardt, Hvidkjaer and Swaminathan, 2005).\(^7\) Third, the index consists of most liquid bonds. This mitigates the infrequent trading problem and provides an upper bound for inferring bond return predictability. Finally, using the index data bypasses the aggregation problem when summarizing test results across individual bonds to draw an unbiased statistical inference.\(^8\)

Figure 1 plots the return series and histograms of the data. Returns of corporate bonds are substantially less volatile than stock and index futures returns. Volatility of corporate bonds is relatively low with an exception for the recent financial crisis period. Daily returns are centered on zero with occasional spikes. There are significant volatility clusterings and distributions of returns clearly deviate from normal for both stocks and bonds.

[Insert Figure 1]

Panel A of Table 1 provides summary statistics for all return series. Over the sample

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\(^6\) The NASD-Bloomberg indices reflect actual transactions throughout the most actively traded portion of the corporate bond market where 65 percent of that activity occurs at the retail level. All bonds included in the basket must have traded on average at least 3 times per day with at least one trade on 80% of the 60 trading days prior to the rebalance date and have a total issued amount of outstanding reported publicly. The index values are calculated at 5:15 pm each day using TRACE transactions. The indices are rebalanced on a monthly basis.

\(^7\) Sarig and Warga (1989) indicate that matrix prices are problematic.

\(^8\) Asness, Moskowitz and Pedersen (2009) use aggregate data and find higher statistical power.
period, the high-yield bond index has the highest mean daily return (0.025%), and the S&P 500 stock index futures has the lowest mean return which is close to zero. The investment-grade bond index return has the lowest daily volatility (0.264%) while the stock index return has the highest volatility (1.317%). The return-risk tradeoff measured by the ratio of mean return to standard deviation is highest for investment-grade bonds (0.064), followed by high-yield bonds (0.041), the stock index (0.005) and index futures (-0.002). Historically, returns of corporate bonds vary from one period to another (see Blume, Keim and Patel, 1991). Over our sample period, corporate bond returns are higher than stock returns partly due to the equity market slump in 2002 and the severe downturn in the recent financial crisis. High-yield bond index returns are negatively skewed and have high kurtosis. Investment-grade bond index returns are also negatively skewed and have kurtosis but the magnitude is milder compared to high-yield bonds. Consistent with previous findings, daily returns of S&P 500 spot and futures indices exhibit kurtosis. Excessive kurtosis is partly attributed to extreme movements in stock and bond prices during the recent financial market turmoil.

Panel B of Table 1 shows that returns are positively contemporaneously correlated across securities. The positive correlation between corporate bond and S&P 500 index returns is consistent with the negative contemporary correlation between yield changes of corporate bonds and stock returns documented by Kwan (1996). Correlation between investment-grade bond and stock index returns is low (0.026), compared to that between high-yield bond and stock index returns (0.315). Cash flows of investment-grade bonds are relatively stable and thus their prices are less sensitive to firm-specific earnings news. On the other hand, speculative bond prices are sensitive to news about firms’ earnings, similar to stocks, because default risk is high. This may explain the difference between these contemporaneous correlation coefficients. The bottom row reports the correlation between returns of stock index futures and other indices. Correlations between bond returns and stock index future returns are somewhat higher than those between bond returns and stock index returns. Not surprisingly, the contemporary correlation between S&P 500 index and stock futures index returns is close to one.
4.2 Tests of serial dependence in conditional mean

Panel C of Table 1 reports autocorrelations of returns. The Ljung-Box (LB) statistics are significant up to a lag order of $p = 10$ for all return series. Bond index returns tend to have positive autocorrelations. The first-order autocorrelation of bond returns is strongly positive but that of stock returns is negative, and the former is much higher than the latter in terms of absolute magnitude. In addition, autocorrelation of high-yield bond returns ($r_{nbbh}$) is higher than that of investment-grade bond returns ($r_{nbbi}$). These results imply that past returns would have higher predictive ability for future returns of corporate bonds than for stocks, and the predictability would also be higher for future returns of high-yield bonds than for future returns of investment-grade bonds.

Panel D of Table 1 shows cross-correlations among returns at different lags. Cross autocorrelation concentrates on the left-hand corner of the table, indicating that cross autocorrelations from stocks to bonds are much higher than from bonds to stocks. Furthermore, the cross autocorrelation from stocks to high-yield bonds is higher than that from stocks to investment-grade bonds, suggesting that positive stock returns lead to positive bond returns in the next day. Investment-grade bond returns are also positively cross-serially correlated with high-yield bond returns.

Table 2 reports results of variance ratio tests and generalized spectral tests. Variance ratio tests account for conditional heteroskedasticity. As shown, all return series fail the variance ratio test (see the left panel), consistent with the finding in Table 1 that returns are significantly serially correlated. The significance level of rejection associated with the variance ratio test is however lower than that of the LB test. One possible reason for the higher rejection level in the LB test is that conditional heteroskedasticity is unaccounted. Variance ratio test statistics are much higher for high-yield bonds, suggesting the existence of a more complicated serial dependence structure for this bond market segment.

The right panel of Table 2 reports the results of MDS tests using the generalized spectral test of Hong (1999). Results overwhelmingly reject the hypothesis of no serial dependence in corporate bond and stock returns. The generalized spectral test can detect a wide range of model misspecifications in mean and are robust to conditional
heteroskedasticity and higher order time-varying moments of unknown form. A distinct advantage is its ability to uncover both linear and nonlinear serial dependence in the moments of any order. Results in Table 2 confirm that the generalized spectral test indeed has more power than the variance ratio test as it rejects the hypothesis of serial independence more strongly. The generalized spectral test rejects the independence hypothesis for index futures returns even at long lags. Test statistics are the highest for high-yield bonds, followed by investment-grade bonds, stocks and index futures. Results suggest the existence of a more complicated dependence structure. We next examine if there exists a component of nonlinear dependence in returns.

[Insert Table 2]

Although tests in Table 2 consider the effect of conditional heteroskedasticity, it remains unclear whether serial dependence is caused by linear and nonlinear factors. To check whether serial dependence contains nonlinearity, we can remove the linear component of returns from an ARMA model and examine whether the adjusted return can pass the variance ratio and generalized spectral tests. If the adjusted return passes the variance ratio test, this implies that all linear relations have been purged. The generalized spectral test will then tell us more about the possibility of nonlinear dependence. A rejection by the generalized spectral test points to the existence of nonlinearity in return dependence.

To remove the linear component of serial dependence, we estimate the following ARMA model:

\[ r_t = \alpha + \sum_{j=1}^{p} \beta_j r_{t-j} + \sum_{i=1}^{q} \gamma_i e_{t-i} + \varepsilon_t \]

where lag orders \( p \) and \( q \) are determined by the BIC information criterion. The BIC criterion provides a consistent order selection for a weakly stationary linear process (see Hannan, 1980). After estimating the model, we remove the AR component from returns and apply the variance ratio test and the generalized spectral test to the linearly adjusted return to check if there is still significant serial dependence.

[Insert Table 3]

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9 The BIC criterion suggests an AR (2) process for the high-yield bond, and an AR (1) process for the investment-grade bond, S&P 500 stock index, and index futures return series.
Table 3 reports results of tests for the adjusted return. After removing the linear return component, we find that all residual returns pass the variance ratio test (see the left panel). This finding suggests that the linear dependence in returns has been successfully purged. However, all adjusted returns fail the generalized spectral test. Thus, although there is no evidence against the white noise hypothesis (see the left panel), the generalized spectral test rejects the MDS hypothesis. Results strongly suggest that nonlinear dependence exists in returns, which cannot be detected by the variance ratio test. These findings demonstrate the power of the generalized spectral test in detecting more complicated serial dependence structure in returns. The structure of serial dependence appears to be more complex for high-yield bonds as exhibited by the relatively high test statistics for this group.

4.3 Linear Granger causality tests

We next turn to Granger-causality tests for the lead-lag relationship between the two markets. We first estimate the VAR model that include returns of high-yield and investment-grade bonds and stocks where the lag order is determined by the BIC criterion. Table 4 reports the results for the vector autoregressive model. The return of the high-yield bond index is significantly related to the lagged-one return (0.145) of the investment-grade bond and the lagged-one return (0.152) of the stock index at the one percent level. The return of the investment-grade bond is significantly related to the lagged-one returns of the high-yield bond (-0.030) and stock (0.050) at the five and one percent levels, respectively. The coefficient of the lagged-one stock index return in the investment-grade bond return equation is much lower than that (0.152) in the high-yield bond equation, indicating that high-yield bond returns are more sensitive to lagged stock market returns. On the other hand, the return of the S&P 500 index is only significantly related to its own return at lag one (-0.166) but significantly related to both investment-grade (0.147) and high-yield bond returns (-0.663) at lag two. Intercept estimates are 0.018, 0.014 and 0.018 for the high-yield bond, investment-grade bond and S&P 500 index return equations, respectively and only the second term is significantly different from zero at the five percent level.

[Insert Table 4]

Results of linear Granger-causality tests in Table 5 show that the high-yield bond
return is Granger-caused by the investment-grade bond return, whereas the investment-grade bond return is not Granger-caused by the high-yield bond return. Both high-yield and investment-grade bond returns are Granger-caused by stock returns at the one percent level. Results also show that stock returns are Granger-caused by both high-yield and investment-grade bond returns but test statistics are weaker.

[Insert Table 5]

4.4 Nonlinear Granger causality tests with heteroskedasticity

Standard Granger causality tests assume that the residual terms in the VAR model are conditionally homoskedastic. If this assumption is violated, causality tests are biased and one may obtain spurious correlations between variables in fitting supposedly uncorrelated data with conditional heteroskedasticity to the linear VAR model.

To check the robustness of standard Granger-causality tests, we conduct the VAR test by accounting for conditional heteroskedasticity. We estimate the following bivariate VAR(1) –GARCH (1, 1) model for each pair of security returns:

\[ Z_t = \delta + \Phi_1 Z_{t-1} + \varepsilon_t \]

where \( \delta = [\delta_1, \delta_2]^T \), \( \Phi_1 = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \), \( \varepsilon_t | I_{t-1} \sim N(0, H_t) \), and the conditional variance-covariance matrix \( H_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix} \) is assumed to have a BEKK representation.

Table 6 reports results of the bivariate VAR model with conditional heteroskedasticity. The effect of the lagged stock return on the high-yield bond return remains significant, even after accounting for the GARCH effect. By contrast, there is no evidence that corporate bond market returns can predict stock market returns. For the high-yield bond regression, the lagged-one coefficient of the stock index return is 0.080, which is significant at the one percent level, whereas for the investment-grade bond regression, the lagged-one coefficient (0.014) of the stock index return is significant at the five percent level. Results again show that high-yield bonds are more closely linked to stocks.

[Insert Table 6]
Results of the causality test under conditional heteroskedasticity are reported in Table 7 where the lag order \( p \) of the VAR is determined by the BIC for each pair of return series. High-yield bond returns are Granger-caused by investment-grade bond returns at the five percent significance level, and investment-grade bond returns are Granger-caused by high-yield bond returns at the one percent level. Both investment-grade and high-yield bond returns are Granger-caused by stock returns at the one percent level. But stock returns are neither Granger-caused by high-yield nor by investment-grade bond returns at the five percent level. This finding contrasts with the result in Table 5 which assumes conditional homoskedasticity. It appears that results in Table 5 are spurious due to ignorance of heteroskedasticity. More robust results in Table 7 suggest that the stock market return leads the bond market return but not vice versa.

[Insert Table 7]

4.5 The GARCH-M effect

Previous studies have documented an existence of the GARCH-M effect in the stock market (see Hamao, Masulis and Ng, 1990). We next examine whether there exists a similar GARCH-M effect in the corporate bond market. Table 8 reports results for the bivariate VAR(1)-GARCH(1,1)-M model. For convenience, the estimate of the GARCH-M response coefficient matrix \( \Psi_1 \) is transposed. For instance, in the high-yield bond index return equation, the coefficient of its own variance is 0.002, the coefficient of the covariance term between high-yield and investment-grade bond index returns is 0.065 and the coefficient of the variance of investment-grade bond index return is 0.158. Overall, we find that the GARCH-M effect is modest. Only the high-yield bond return is barely significantly related to volatility of the investment-grade bond index return (0.158) at the five percent level, while the rest of the GARCH-M coefficients are insignificantly different from zero. There is no evidence that volatility of corporate bond returns affects stock returns. The cross dependence of conditional mean corporate bond returns on past stock returns remains unchanged. Results continue to show that the stock market return predicts the corporate bond market return.

[Insert Table 8]
4.6 Cross dependence in excess returns

The total return of a risky asset can be divided into a riskfree interest component which represents a compensation for the investor’s time preference and an excess return component, a reward for bearing risk. If the stock market is more efficient in impounding the firm’s cash flow information than the corporate bond market, the excess stock return should lead the excess corporate bond return. Focusing on excess returns allows us to net out the effect of interest rates. As such, causality tests on excess returns are expected to be more revealing. We next examine the relationship between the excess returns of corporate bonds and stocks.

The riskfree rate data were downloaded from Ken French’s website.\textsuperscript{10} We subtract the riskfree rate from the return of risky securities and conduct the Granger causality test on excess returns.\textsuperscript{11} Table 9 reports test results which take into account conditional heteroskedasticity and the lag orders are determined by BIC. Results show that high-yield bond excess returns are Granger-caused by investment-grade bond excess returns, and vice versa at the five and one percent levels, respectively. High-yield and investment-grade bond returns thus exhibit a feedback relation.

High-yield bond excess returns and investment-grade bond excess returns are Granger-caused by stock excess returns at the one percent significance level. By contrast, stock excess returns are neither Granger-caused by high-yield bond nor by investment-grade bond excess returns at the five percent level. Results continue to support the hypothesis that the stock market leads the corporate bond market even after controlling for the interest rate effect.

\[\text{[Insert Table 9]}\]

4.7 Tests using stock index futures data

Previous studies have found that the futures market leads the spot market (see, for example, Stoll and Whaley, 1990; Chan, 1992; Miller, Muthuswamy and Whaley, 1994; Wu, Li and Zhang, 2005). We next replace the S&P 500 stock index return by the S&P 500 index futures return in the causality test. The S&P 500 futures return is driven by

\textsuperscript{10} The same website provides the Fama-French factors (see Fama and French, 1993, 1996).
\textsuperscript{11} We also estimate the VAR-GARCH-M models for these excess returns and get similar results. These results are available upon request.
similar fundamental factors that affect the stock index return. Nevertheless, if the index futures market impounds information more quickly, we should observe a stronger predictive effect on corporate bond returns.

Panel A of Table 10 reports results of causality tests using the S&P 500 futures and corporate bond returns. These tests account for conditional heteroskedasticity in returns. Results show that both high-yield and investment-grade bond returns are significantly Granger-caused by the S&P 500 futures return at the one percent level, whereas the S&P 500 futures return is neither Granger-caused by high-yield nor by investment-grade bond returns at the five percent level. The predictive relationship from the index futures return to the high-yield bond return is stronger than that based on the stock index return (see Table 7). Results strongly support the hypothesis that corporate bond returns are predicted by past stock index futures returns.

We next extend the test on the excess returns by including index futures returns. Panel B of Table 10 reports results of causality tests. Results are even stronger after controlling for the effect of interest rates. Both high-yield and investment-grade bond returns are Granger-caused by stock index futures returns. By contrast, index futures returns are not Granger-caused by either investment-grade or high-yield bond returns. Results suggest that the causal relation is primarily driven by the response to the flow of fundamental information rather than the discount rate.

5. Effects of Government Bond and Stock Returns on Corporate Bond Returns

Corporate bonds can be perceived as a hybrid of riskless bonds and stocks and so their returns are expected to be related to both government bond and stock returns. These relations depend on the risk of corporate bonds. High-yield bonds have high default risk, which makes their expected cash flows tied more closely to firm value changes. As such, returns of high-yield bonds are more sensitive to news about firms’ earnings and economic fundamentals. As variations of stock returns are closely linked to changes in

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12 Kawaller, Koch and Koch (1987) indicate that “the conventional wisdom among professional traders is that movements in the S&P500 futures price reflect market expectations of subsequent movements in cash prices. The futures price presumably embodies all available information regarding events that will affect cash prices and responds quickly to new information.” If so, the futures market may contain more information.

13 For robustness check, we also estimate the bivariate VAR-GARCH(1,1)-M model using futures data. The results are consistent with the Granger-causality test. These results are available upon request.
firms’ expected future cash flow, the correlation will be high between high-yield bond and stock returns. Thus, high-yield bonds resemble stocks more than riskless bonds (see Blume, Keim and Patel, 1991; Cornell and Green, 1991; and Kwan, 1996). On the other hand, investment-grade bonds have relatively stable expected cash flows because their default risk is small. This makes returns of investment-grade bonds less sensitive to news about firms’ future cash flows. Instead, these safe bonds are more sensitive to interest rate changes and behave more like government bonds. Therefore, one would expect higher contemporaneous and lagged correlations between investment-grade bond and government bond returns.

We test these hypotheses using the following time-series regression:

\[ r_t = \alpha + \sum_{j=1}^{q_1} \beta_j^{AR} r_{-j} + \sum_{j=0}^{q_2} \beta_j^S r_{-j}^{SP500} + \sum_{j=0}^{q_3} \beta_j^L r_{-j}^{LEHMAN} + \varepsilon_t \]

where the dependent variable \( r_t \) is either the high-yield bond return \( r_{nbbh_t} \) or the investment-grade bond return \( r_{nbbi_t} \), \( \beta^{AR} \) is the autoregressive coefficient, \( \beta^S \) is the response coefficient associated with the S&P 500 index return, \( \beta^L \) is the coefficient of the Lehman intermediate government bond index return, and \( q_1, q_2, q_3 \) denote the lag orders for corporate bond, stock and government bond returns, respectively.\(^{14}\) We incorporate current and lagged stock index returns and Lehman intermediate government bond index returns to capture both contemporaneous and lagged effects on corporate bond returns.

The above regression ignores the fact that investment- and speculative-grade bond returns are cross autocorrelated (see Panel D of Table 1 and Table 4). To account for this dynamic relation, we estimate the following time-series regression:

\[ r_t = \alpha + \sum_{j=1}^{q_1} \beta_j^{AR} r_{-j} + \sum_{j=0}^{q_2} \beta_j^S r_{-j}^{SP500} + \sum_{j=0}^{q_3} \beta_j^L r_{-j}^{LEHMAN} + \sum_{j=1}^{q_4} \beta_j^C r_{-j}^{C} + \varepsilon_t \]

where \( r_{-j}^{C} \) is the other corporate bond index return, i.e., if the dependent variable \( r_t = r_{nbbh_t} \), then \( r_t^{C} = r_{nbbi_t} \), and vice versa, and \( q_4 \) is the lag order of the cross serial correlation term. Here we add the lagged investment-grade bond returns in the

\(^{14}\) The Lehman intermediate government bond index is downloaded from Datastream.
high-yield bond return regression, and the lagged high-yield bond returns in the investment-grade bond return regression to capture the cross autocorrelation between these two bond market segments.

Table 11 reports results for these regressions. The BIC tests reveal that a lag order of 4 is adequate to capture the lagged effects of regressors but we also examine the sensitivity of results to different lag orders. Conditional heteroskedasticity is taken into account in all regressions. Results show a significant positive effect of government bond returns on both high-yield and investment-grade bond returns. The effect of government bond returns, as measured by the sum of response coefficients $\sum \beta_j^t$, is significant at the one percent level for both high-yield and investment-grade bonds. But the magnitude of the effect of government bond returns on investment-grade bonds is three times as big compared to that on high-yield bonds. The result shows that investment-grade bonds are much more sensitive to riskfree rate changes.

The effect of investment-grade bond returns on high-yield bond returns is not significant. By contrast, the effect of high-yield bond returns on investment-grade bond returns is significant at the one percent level after controlling for the effects of government bond and stock index returns and the lagged effects of investment-grade bond returns.

The effect of stock index returns on high-yield bond returns is significantly positive, confirming that high-yield bond returns are affected by risk factors similar to those driving stock returns. This positive relationship remains quite significant even after including lagged investment-grade bond returns in the regression. In comparison, the effect of stock index returns on investment-grade bond returns is much smaller although it is still significantly positive. Tests reject the hypothesis of no stock return effect regardless of whether we include the contemporaneous stock index returns or not (see the test results in the last two columns). In sum, stock returns predict corporate bond returns, and this predictive relationship is stronger for high-yield bonds. Moreover, investment-grade bonds behave more like government bonds whereas high-yield bonds behave more like stocks.

Empirical results are not sensitive to the lag order for the time-series regression. Panel
B increases the lag order by one for each explanatory variable while Panel C reports the results using the differential lag structures for different variables and including only the current return of government bonds in the regression. Results again show that high-yield bonds are more closely associated with stocks and investment-grade bonds are more like government bonds. Both high-yield and investment-grade bond returns are significantly related to concurrent and lagged stock market returns. Our results for investment-grade bonds echo the findings of Kwan (1996) while our results for high-yield bonds are consistent with the findings of Downing et al. (2009) based on individual bond data. Downing et al. (2009) show that the lead-lag relation primarily holds for individual convertible bonds with conversion options deep in the money and bonds with severe financial distress. By contrast, using the broad-based market index data, we find that stock market returns lead both investment- and speculative-grade straight (nonconvertible) bond returns.

6. Conclusion

An issue central to financial research is the predictability of asset returns. There is mounting evidence that stock returns are predictable. An important question is whether returns are also predictable for other asset classes. This paper examines this issue for the corporate bond market and employs empirical methodologies that are robust to nonlinearity in serial dependence and conditional heteroskedasticity.

Empirical evidence strongly suggests that corporate bond market returns are predictable. There is clear evidence of return predictability for both investment-grade and high-yield straight bonds. These results are robust to alternative model specifications and return measures. Results show that corporate bond market returns exhibit higher autocorrelation and a more complicated structure of serial dependence than stock market returns. Stock market returns lead both high-yield and investment-grade bond returns whereas there is little evidence that corporate bond market returns lead stock market returns. This lead-lag relation is stronger between high-yield bond returns and stock returns. The lead-lag relation is more pronounced when we use stock index futures data to represent the stock market activity and control the effect of interest rates in the causality
test. Considering the effects of government bond returns and cross dependence in different bond market segments does not alter the conclusion for the lead-lag relation between the stock and bond markets.

Our findings provide important implications for corporate bond modeling and asset pricing tests. Results suggest that tests of the risk-return tradeoff in corporate bonds should take into account the cross-serial dependence between bond and stock returns. Our findings also impose restrictions on the specification of the term structure model. In particular, our results suggest that the term structure model of defaultable bonds should account for the serial and cross-serial dependence in corporate bond price changes in order to provide more satisfactory explanation for corporate bond yields.

The cause for predictability of corporate bond returns is not immediately clear. Return predictability could be due to bond illiquidity, transaction cost, market structure or other frictions. An exploration of the cause for our results is an important extension of this paper and we leave this for a future study.
Appendix

In this appendix, we describe the generalized spectral test of Hong (1999) and Hong and Lee (2005, 2007). Assume that a time series \{X_t\} follows the process \[ X_t = g(I_{t-1}, \theta) + \varepsilon_t, \] where \( I_{t-1} \) is the information set at time \( t-1 \), \( g(I_{t-1}, \theta) \) is conditional mean \( E(Y_t | I_{t-1}) \), and \( \theta \in \Theta \) is a parameter value in a limited dimensional parameter set \( \Theta \). In the present case, \( X_t \) can be the return on a stock or corporate bond index, and \( g(I_{t-1}, \theta) = \mu \).

Denote the demeaned return series as \( \varepsilon_t \equiv X_t - \mu \). Under the null hypothesis \( H_0 \) of \( E[\varepsilon_t | I_{t-1}] = 0 \), this implies that \( E[\varepsilon_t | I_{t-1}] = 0 \), \( I_{t-1}^\varepsilon \equiv \{\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots\} \), and the return is unpredictable because the demeaned series \( \{\varepsilon_t\} \) is a martingale difference sequence (MDS), where its conditional mean does not exhibit any serial dependence.

Suppose that \( \{\varepsilon_t\} \) is a strictly stationary process with a marginal characteristic function \( \varphi(u) = E(e^{iux}) \) and a pairwise joint characteristic function \( \varphi_j(u,v) = E(e^{iuX_t+ivX_{t-j}}) \) where \( i = \sqrt{-1} \), \( -\infty < u, v < \infty \), and \( j = 0, \pm 1, \pm 2, \ldots \). The generalized spectral test of Hong (1999) uses the spectrum of the transformed series \( \{e^{iux}\} \) in the MDS test. The generalized spectral density function can be written as

\[
f(\omega, u, v) = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \sigma_j(u,v)e^{-j\omega} , \omega \in [-\pi, \pi] , -\infty < u, v < \infty
\]

where \( \omega \) is the frequency, and \( \sigma_j(u,v) = \text{cov}(e^{iuY_t}, e^{ivY_{t-j}}) = \varphi_j(u,v) - \varphi(u)\varphi_{t-j}(v) \) is the covariance function of the transformed series. This generalized spectral density function can capture any form of pairwise serial dependence in \( \{\varepsilon_t\} \), i.e., dependence between \( \varepsilon_t \) and \( \varepsilon_{t-j} \) at any order \( j \neq 0 \), including the first moment dependence (mean dependence) and higher-order moments dependence, such as volatility, skewness and kurtosis dependence. This is because \( \sigma_j(u,v) = 0 \) if and only if \( \varepsilon_t \) and \( \varepsilon_{t-j} \) are independent. In addition, this function does not need any information for the moment
conditions of \( \{ \varepsilon_t \} \).

If the second-order moments \( E(\varepsilon_t^2) \) exists, we can obtain the power spectrum as the derivative of \( f(\omega, u, v) \) at \((u, v) = (0, 0)\):

\[
\frac{\partial^2}{\partial u \partial v} f(\omega, u, v) \bigg|_{(u, v) = (0, 0)} = -\frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \text{cov}(\varepsilon_t, \varepsilon_{t-j}) e^{-j\omega} = -h(\omega), \quad \omega \in [-\pi, \pi].
\]

Power spectrum is a common analytical tool in time series for describing the linear serial correlation (Priestley, 1981) and for this reason \( f(\omega, u, v) \) is called the general spectrum of \( \{ \varepsilon_t \} \). In the literature, the power spectrum \( h(\omega) \) has been used widely in analyzing cyclical economic fluctuations. However, \( h(\omega) \) can only describe the linear relationship, which may not be adequate for capturing more subtle economic phenomena. By contrast, the generalized spectrum can capture cyclical dynamics caused not only by linear but also by nonlinear dependence, including the serial dependence in volatility, skewness and any other higher-order conditional moments. Therefore, it is more general than the power spectrum \( h(\omega) \).

However, the generalized spectrum \( f(\omega, u, v) \) itself is not suitable for testing \( H_0 \) since it deals primarily with the issue of whether \( \{ \varepsilon_t \} \) is an i.i.d. series, which involves all pairwise serial dependencies in \( \{ X_t \} \) at various lags. Under \( H_0 \) we only need to test whether the conditional mean of \( \varepsilon_t \) is serially dependent or \( \{ \varepsilon_t \} \) is a martingale difference sequence or not. When \( \{ \varepsilon_t \} \) is not a Gaussian process, there are differences between a white noise and an MDS. Suppose that \( \{ \varepsilon_t \} \) is an MDS or \( E(\varepsilon_t | I_{t-1}) = 0 \), but it exhibits volatility clustering effect, i.e., \( \text{cov}(\varepsilon_t^2, \varepsilon_{t-1}^2) \neq 0 \). In this case, \( H_0 \) holds. But the test based on the generalized spectral function will reject the hypothesis that \( \{ \varepsilon_t \} \) is i.i.d although it is an MDS.

To cope with this problem, Hong and Lee (2005, 2007) propose the use of the partial derivative of the generalized spectral function in the MDS test, which focuses on the
serial dependence of conditional mean. Since the partial derivative function is not affected by serial dependence of higher-order moments, it is suitable for testing the MDS hypothesis. The partial derivative of the generalized spectral function can be written as

$$f^{(0,1,0)}(\omega,u,v) = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \sigma_j^{(1,0)}(u,v)e^{-j\omega u}, \omega \in [-\pi, \pi], -\infty < v < \infty,$$

where

$$\sigma_j^{(1,0)}(0,v) = \frac{\partial}{\partial u} \sigma_j(u,v) \bigg|_{u=0} = \text{cov}(i\varepsilon, e^{j\omega - j\varepsilon}).$$

The covariance measure $\sigma_j^{(1,0)}(0,v)$ checks whether the autoregression function $E(\varepsilon_t | \varepsilon_{t-j}) = 0$ is zero at lag $j$. An important property is that $\sigma_j^{(1,0)}(0,v) = 0$ for all $v \in R$ if and only if $E(\varepsilon_t | \varepsilon_{t-j}) = 0$ a.s. Therefore, it can detect any linear and nonlinear serial dependence in mean, including the process with zero autocorrelation. An example is the nonlinear moving average process $\varepsilon_t = \alpha z_{t-1}z_{t-2} + z_t, \{z_t\} \sim i.i.d.(0,\sigma^2)$. This process is a white noise but not an MDS, because the conditional mean is time-varying and dependent on past values. In this case, using the partial derivative of the generalized spectrum allows us to reject the MDS hypothesis because $E(\varepsilon_t | \varepsilon_{t-j}) \neq 0$, whereas the test based on the autocovariance function $\text{cov}(\varepsilon_t, \varepsilon_{t-j})$ will miss this subtle nonlinear dependent process.

Under $H_0$, the generalized spectral partial derivative function $f^{(0,1,0)}(\omega,0,v)$ becomes a “flat” spectrum,

$$f^{(0,1,0)}(\omega,0,v) = f_0^{(0,1,0)}(\omega,0,v) = \frac{1}{2\pi} \sigma_0^{(1,0)}(0,v), \omega \in [-\pi, \pi], -\infty < v < \infty,$$

We can estimate the generalized spectral partial derivative $f^{(0,1,0)}(\omega,0,v)$ and check whether it is a flat spectrum. If it is, then $H_0$ is true; otherwise the MDS hypothesis is rejected. Thus, we can test serial dependence of returns easily based on the generalized spectral test.

The generalized spectral test has several advantages over conventional serial correlation tests. Using the partial derivative function $f^{(0,1,0)}(\omega,0,v)$, one can test whether there exists serial dependence in mean or not where the dimension of the information set...
can be infinite. Because \( f^{(0,1,0)}(\omega, 0, v) \) is always flat no matter whether higher-order conditional moments are changing over time or not, one can construct a test for the MDS that is robust to time-varying higher-order moments of any unknown form. Also, the test method can be used to check a large number of lags and it naturally discounts higher order lags.

To test the MDS hypothesis, one can compare the consistent estimators for \( f^{(0,1,0)}(\omega, 0, v) \) and \( f^{(0,1,0)}_0(\omega, 0, v) \). Hong and Lee (2005, 2007) propose a test statistic to examine whether \( \hat{\varepsilon}_i = X_i - g\left(I_{t-1}, \hat{\theta}\right) \) satisfies the condition that \( E(\hat{\varepsilon}_i | I_{t-1}) = 0 \). They show that the generalized spectral partial derivative \( f^{(0,1,0)}(\omega, 0, v) \) can be consistently estimated by a smoothed kernel,

\[
\hat{f}^{(0,1,0)}(\omega, 0, v) = \frac{1}{2\pi} \sum_{j=1}^{T-1} (1 - |j| / T)^{1/2} k(j / p) \hat{\rho}^{(1,0)}_j(0, v) e^{-j\omega}, \omega \in [-\pi, \pi], -\infty < v < \infty
\]

where

\[
\hat{\rho}^{(1,0)}_j(0, v) = (T - |j|)^{-1} \sum_{t=|j|+1}^{T} i \hat{\varepsilon}_t \hat{h}_{t-|j|}(v), j = 0, \pm 1, \ldots, \pm (T-1),
\]

\[
\hat{h}_{t-|j|}(v) = \hat{\phi}_{t-|j|}(v) - \hat{G}_j \hat{\beta}_j(v), \hat{G}_j = \frac{\partial}{\partial \theta} g(I_{t-1}, \hat{\theta}),
\]

\[
\hat{\beta}_j(v) = \left( \sum_{t=|j|+1}^{T} \hat{G}_j \hat{\phi}_t \right)^{-1} \sum_{t=|j|+1}^{T} \hat{G}_j \hat{\phi}_{t-|j|}(v),
\]

\( p = p(T) \) is a bandwidth, and \( k : (-\infty, \infty) \rightarrow [-1,1] \) is a symmetric kernel with \( k(0) = 0 \).

An example of \( k(\cdot) \) is the Bartlett kernel function, i.e., \( k(z) = (1 - |z|)I(|z| \leq 1) \), where \( I(\cdot) \) is an indicator function which equals to one when \( z \) is between \([-1, 1]\) and zero, otherwise.

The flat spectrum can be estimated by

\[
\hat{f}^{(0,1,0)}_0(\omega, 0, v) = \frac{1}{2\pi} \hat{\rho}^{(1,0)}_0(0, v), \omega \in [-\pi, \pi], -\infty < v < \infty.
\]

Under \( H_0 \), \( f^{(0,1,0)}(\omega, 0, v) = f^{(0,1,0)}_0(\omega, 0, v) \). Therefore, when \( \{\hat{\varepsilon}_i\} \) is an MDS, the estimated function \( \hat{f}^{(0,1,0)}(\omega, 0, v) \) should be very close to \( \hat{f}^{(0,1,0)}_0(\omega, 0, v) \). If \( \hat{f}^{(0,1,0)}(\omega, 0, v) \) turns out to be quite different from \( \hat{f}^{(0,1,0)}_0(\omega, 0, v) \), then \( \{\hat{\varepsilon}_i\} \) is not an MDS. Based on this
relationship, Hong and Lee (2005, 2007) construct the following test statistic:

\[
M(p) = \left[ \frac{1}{T} \sum_{j=1}^{T} k^2(j/p)(T-j) \right] \left| \hat{\rho}_j^{(0,0)}(0,0)^2 dW(v) - \hat{C}_j(p) \right| \sqrt{\hat{D}_j(p)}
\]

where

\[
\hat{C}_j(p) = \hat{S}^2 \int \left[ \frac{1}{T} \sum_{t=1}^{T} \hat{h}_i(v) \right]^2 dW(v) \sum_{j=1}^{T} k^2(j/p),
\]

\[
\hat{D}_j(p) = 2\hat{S}^2 \int \left[ \frac{1}{T} \sum_{t=1}^{T} \hat{h}_i(u) \hat{h}_i(v) \right]^2 dW(u)dW(v) \sum_{j=1}^{T} k^4(j/p),
\]

\[
\hat{S}^2 = \frac{1}{T} \sum_{t=1}^{T} \hat{e}_t^2,
\]

and \( W(\cdot) : R \to R^+ \) is a nondecreasing function which sets weights symmetrically around zero. An example of \( W(\cdot) \) is the \( N(0,1) \) cumulative density function commonly used in the characteristic function literature. Hong and Lee (2005) show that \( M(p) \xrightarrow{d} N(0,1) \) under \( H_0 \). Otherwise, \( M(p) \) goes to positive infinity as the sample size increases. Thus, if \( M(p) \) is greater than the critical value of \( N(0,1) \) at a significant level, the difference between \( \hat{f}^{(0,1,0)}(\omega,0,\nu) \) and \( \hat{f}_0^{(0,1,0)}(\omega,0,\nu) \) is significant and the MDS hypothesis is rejected.

The steps for carrying out the generalized spectral test of the MDS hypothesis can be summarized as follows.

1. Estimate \( \hat{\mu} \) and compute the error \( \{\hat{e}_t\} \equiv y_t - \hat{\mu} \).
2. Calculate the derivative \( \hat{G}_t = \frac{\partial}{\partial \theta} g(I_{t-1}, \hat{\theta}) = 1 \).
3. Regress \( \hat{\phi}_{t-1}\cdot(v) \) on \( \hat{G}_t \) and compute the residual \( \hat{h}_{t-1}\cdot(v) \). We separately regress the real and imaginary components of \( \hat{\phi}_{t-1}\cdot(v) \) on \( \hat{G}_t \).
4. Compute the test statistic \( M(p) \).
5. Compare \( M(p) \) with the critical value of standard normal distribution \( N(0,1) \) at a significance level. For example, the critical value at the 1% level is 2.33. If the test statistic is greater than this critical value, the hypothesis \( H_0 \) is rejected at the 1% level, and we conclude that the series is not an MDS.
References


### Table 1
#### Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean (%)</th>
<th>Standard Deviation (%)</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{nbbh}$</td>
<td>0.025</td>
<td>0.617</td>
<td>-1.852</td>
<td>48.737</td>
</tr>
<tr>
<td>$r_{nbii}$</td>
<td>0.017</td>
<td>0.264</td>
<td>-0.720</td>
<td>27.159</td>
</tr>
<tr>
<td>$r_{sp500}$</td>
<td>0.006</td>
<td>1.317</td>
<td>-0.275</td>
<td>16.758</td>
</tr>
<tr>
<td>$r_{futures}$</td>
<td>-0.003</td>
<td>1.295</td>
<td>0.076</td>
<td>23.796</td>
</tr>
</tbody>
</table>

#### Panel B: Contemporaneous correlation

<table>
<thead>
<tr>
<th></th>
<th>$r_{nbbh}$</th>
<th>$r_{nbii}$</th>
<th>$r_{sp500}$</th>
<th>$r_{futures}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{nbbh}$</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_{nbii}$</td>
<td>0.470</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_{sp500}$</td>
<td>0.315</td>
<td>0.026</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>$r_{futures}$</td>
<td>0.344</td>
<td>0.056</td>
<td>0.982</td>
<td>1.000</td>
</tr>
</tbody>
</table>

#### Panel C: Autocorrelation

<table>
<thead>
<tr>
<th>Lag</th>
<th>$r_{nbbh}$</th>
<th>$r_{nbii}$</th>
<th>$r_{sp500}$</th>
<th>$r_{futures}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P=1</td>
<td>0.387***</td>
<td>0.279***</td>
<td>-0.133***</td>
<td>-0.115***</td>
</tr>
<tr>
<td></td>
<td>(235.02)</td>
<td>(122.82)</td>
<td>(27.781)</td>
<td>(20.654)</td>
</tr>
<tr>
<td>P=2</td>
<td>0.115***</td>
<td>-0.028***</td>
<td>-0.109***</td>
<td>-0.136***</td>
</tr>
<tr>
<td></td>
<td>(255.00)</td>
<td>(124.02)</td>
<td>(46.835)</td>
<td>(49.639)</td>
</tr>
<tr>
<td>P=3</td>
<td>0.104***</td>
<td>-0.018***</td>
<td>0.097***</td>
<td>0.090***</td>
</tr>
<tr>
<td></td>
<td>(271.07)</td>
<td>(124.55)</td>
<td>(61.043)</td>
<td>(62.292)</td>
</tr>
<tr>
<td>P=4</td>
<td>0.157***</td>
<td>0.017***</td>
<td>-0.059***</td>
<td>-0.046***</td>
</tr>
<tr>
<td></td>
<td>(307.17)</td>
<td>(124.98)</td>
<td>(66.458)</td>
<td>(65.657)</td>
</tr>
<tr>
<td>P=5</td>
<td>0.118***</td>
<td>0.038***</td>
<td>-0.043***</td>
<td>-0.055***</td>
</tr>
<tr>
<td></td>
<td>(327.58)</td>
<td>(127.27)</td>
<td>(69.295)</td>
<td>(70.319)</td>
</tr>
<tr>
<td>P=6</td>
<td>0.062***</td>
<td>0.078***</td>
<td>0.043***</td>
<td>0.055***</td>
</tr>
<tr>
<td></td>
<td>(333.11)</td>
<td>(136.73)</td>
<td>(72.195)</td>
<td>(75.064)</td>
</tr>
<tr>
<td>P=7</td>
<td>0.092***</td>
<td>0.093***</td>
<td>-0.065***</td>
<td>-0.052***</td>
</tr>
<tr>
<td></td>
<td>(345.34)</td>
<td>(150.19)</td>
<td>(78.766)</td>
<td>(79.311)</td>
</tr>
<tr>
<td>P=8</td>
<td>0.021***</td>
<td>0.050***</td>
<td>0.049***</td>
<td>0.052***</td>
</tr>
<tr>
<td></td>
<td>(345.97)</td>
<td>(153.99)</td>
<td>(82.465)</td>
<td>(83.523)</td>
</tr>
<tr>
<td>P=9</td>
<td>0.052***</td>
<td>0.038***</td>
<td>0.005***</td>
<td>-0.027***</td>
</tr>
<tr>
<td></td>
<td>(349.86)</td>
<td>(156.18)</td>
<td>(82.505)</td>
<td>(84.666)</td>
</tr>
<tr>
<td>P=10</td>
<td>0.071***</td>
<td>0.109***</td>
<td>0.022***</td>
<td>0.016***</td>
</tr>
<tr>
<td></td>
<td>(357.19)</td>
<td>(174.08)</td>
<td>(83.219)</td>
<td>(85.061)</td>
</tr>
</tbody>
</table>

#### Panel D: Cross correlation

<table>
<thead>
<tr>
<th></th>
<th>$r_{nbbh}$</th>
<th>$r_{nbii}$</th>
<th>$r_{sp500}$</th>
<th>$r_{futures}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{nbbh}_{t-1}$</td>
<td>0.387</td>
<td>0.170</td>
<td>-0.097</td>
<td>-0.105</td>
</tr>
<tr>
<td>$r_{nbii}_{t-1}$</td>
<td>0.174</td>
<td>0.279</td>
<td>-0.083</td>
<td>-0.102</td>
</tr>
<tr>
<td>$r_{sp500}_{t-1}$</td>
<td>0.398</td>
<td>0.235</td>
<td>-0.133</td>
<td>-0.112</td>
</tr>
<tr>
<td>$r_{futures}_{t-1}$</td>
<td>0.407</td>
<td>0.240</td>
<td>-0.121</td>
<td>-0.115</td>
</tr>
</tbody>
</table>

The Ljung-Box statistics are included in parentheses. *** and ** denote significance at the 1% and 5% levels, respectively. $r_{nbbh}$, $r_{nbii}$, $r_{sp500}$, and $r_{futures}$ represent high-yield, investment-grade, S&P 500 index and index futures returns, respectively.
Table 2
Variance Ratio and MDS Tests for Corporate Bond, Stock and Index Futures Returns

<table>
<thead>
<tr>
<th>Lag (p)</th>
<th>Variance ratio test</th>
<th>Martingale difference series test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r_nbbh</td>
<td>r_nbhi</td>
</tr>
<tr>
<td>1</td>
<td>5.105***</td>
<td>3.107***</td>
</tr>
<tr>
<td>3</td>
<td>4.692***</td>
<td>2.300***</td>
</tr>
<tr>
<td>4</td>
<td>4.700***</td>
<td>2.114***</td>
</tr>
<tr>
<td>5</td>
<td>4.766***</td>
<td>2.023***</td>
</tr>
<tr>
<td>6</td>
<td>4.816***</td>
<td>2.039***</td>
</tr>
<tr>
<td>7</td>
<td>4.896***</td>
<td>2.143***</td>
</tr>
<tr>
<td>8</td>
<td>4.921***</td>
<td>2.255***</td>
</tr>
<tr>
<td>9</td>
<td>4.941***</td>
<td>2.342***</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p* is the lag order used in computing test statistics. Test statistics follow the normal distribution. ***,**, * denote significance at the 1%, 5% and 10% levels, respectively.

Table 3
Variance Ratio and MDS Tests after Adjusting for the Linear Relationship

<table>
<thead>
<tr>
<th>Lag (p)</th>
<th>Variance ratio test</th>
<th>Martingale difference series test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r_nbbh</td>
<td>r_nbhi</td>
</tr>
<tr>
<td>AR(2)</td>
<td>AR(1)</td>
<td>AR(1)</td>
</tr>
<tr>
<td>1</td>
<td>7.146***</td>
<td>2.292**</td>
</tr>
<tr>
<td>2</td>
<td>-0.100</td>
<td>0.353</td>
</tr>
<tr>
<td>3</td>
<td>-0.158</td>
<td>-0.277</td>
</tr>
<tr>
<td>4</td>
<td>-0.075</td>
<td>-0.479</td>
</tr>
<tr>
<td>5</td>
<td>0.070</td>
<td>-0.535</td>
</tr>
<tr>
<td>6</td>
<td>0.232</td>
<td>-0.531</td>
</tr>
<tr>
<td>7</td>
<td>0.322</td>
<td>-0.461</td>
</tr>
<tr>
<td>8</td>
<td>0.467</td>
<td>-0.315</td>
</tr>
<tr>
<td>9</td>
<td>0.533</td>
<td>-0.185</td>
</tr>
<tr>
<td>10</td>
<td>0.599</td>
<td>-0.103</td>
</tr>
</tbody>
</table>

*p* is the lag order used in computing test statistics. Test statistics follow the normal distribution. ***,**, * denote significance at the 1%, 5% and 10% levels, respectively.
Table 4
VAR Estimation of Corporate Bond and Stock Returns

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Granger-causality test</th>
</tr>
</thead>
<tbody>
<tr>
<td>High-yield bond return vs. investment-grade bond return: $p=2$</td>
<td></td>
</tr>
<tr>
<td>Hypothesis 1</td>
<td>4.92***</td>
</tr>
<tr>
<td>Hypothesis 2</td>
<td>2.17</td>
</tr>
<tr>
<td>High-yield bond return vs. stock return: $p=4$</td>
<td></td>
</tr>
<tr>
<td>Hypothesis 3</td>
<td>46.19***</td>
</tr>
<tr>
<td>Hypothesis 4</td>
<td>3.41***</td>
</tr>
<tr>
<td>Investment-grade bond return vs. stock return: $p=2$</td>
<td></td>
</tr>
<tr>
<td>Hypothesis 5</td>
<td>43.22***</td>
</tr>
<tr>
<td>Hypothesis 6</td>
<td>9.38***</td>
</tr>
</tbody>
</table>

Reported are the $F$ statistics for Granger-causality tests. ***,**,* denote significance at the 1%, 5% and 10% levels, respectively. The lag orders ($p$) are selected using the BIC information criterion.

- **Hypothesis 1**: High-yield bond returns are not Granger-caused by investment-grade bond returns.
- **Hypothesis 2**: Investment-grade bond returns are not Granger-caused by high yield bond returns.
- **Hypothesis 3**: High-yield bond returns are not Granger-caused by stock returns.
- **Hypothesis 4**: Stock returns are not Granger-caused by high-yield bond returns.
- **Hypothesis 5**: Investment-grade bond returns are not Granger-caused by stock returns.
- **Hypothesis 6**: Stock returns are not Granger-caused by investment-grade bond returns.
### Table 6

<table>
<thead>
<tr>
<th></th>
<th>$Z_t = [r_{nbbh_t}, r_{nbbi_t}]^T$</th>
<th>$Z_t = [r_{nbbh_t}, r_{sp500_t}]^T$</th>
<th>$Z_t = [r_{nbbi_t}, r_{sp500_t}]^T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0.039*** (6.65) 0.013*** (2.51)</td>
<td>0.034*** (5.60) 0.069*** (3.30)</td>
<td>0.012** (2.35) 0.041* (1.96)</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>0.344*** (12.29) 0.043 (1.22)</td>
<td>0.31*** (11.89) 0.080*** (9.92)</td>
<td>0.18*** (6.21) 0.014** (2.18)</td>
</tr>
<tr>
<td>$c$</td>
<td>0.040*** (5.30) 0.012** (2.03)</td>
<td>-0.042*** (-4.29) -0.020 (-0.36)</td>
<td>0.039*** (3.13) -0.019 (-0.59)</td>
</tr>
<tr>
<td>$a$</td>
<td>0.433*** (9.26) 0.057*** (3.14)</td>
<td>-0.345*** (-7.11) 0.148 (1.56)</td>
<td>-0.283*** (-5.41) -0.40 (-1.56)</td>
</tr>
<tr>
<td>$g$</td>
<td>-0.907*** (-48.10) 0.021*** (3.46)</td>
<td>-0.918*** (-41.46) -0.072(-1.61)</td>
<td>-0.927*** (-38.74) -0.591***(-5.11)</td>
</tr>
</tbody>
</table>

The estimated models are $Z_t = \delta + \Phi Z_{t-1} + \epsilon_t$ where $\delta = [\delta_1, \delta_2]^T$ is the intercept vector, $\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix}$ is the VAR response coefficient matrix, $\epsilon_t | I_{t-1} \sim N(0, H_t)$, and the conditional variance-covariance matrix $H_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix}$ is assumed to have a BEKK representation, i.e.,

$$H_t = c^T c + a^T \epsilon_{t-1} \epsilon_{t-1} a + g^T H_{t-1} g$$

where $c = \begin{bmatrix} c_1 & c_2 \\ 0 & c_{21} \end{bmatrix}$, $a = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $g = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$, $\epsilon_t = \begin{bmatrix} \epsilon_{1,t-1} \\ \epsilon_{2,t-1} \end{bmatrix}$. The t-statistics are included in parentheses. ***, **, * denote significance at the 1%, 5% and 10% levels, respectively. The intercept vector $\delta$ is transposed in the report where the first (second) element is associated with the first (second) dependent variable in the bivariate VAR system.
Table 7
Nonlinear Granger Causality Tests between Corporate Bond and Stock Returns with Conditional Heteroskedasticity

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Granger-causality test</th>
</tr>
</thead>
<tbody>
<tr>
<td>High-yield bond return vs. investment-grade bond return: $p=2$</td>
<td></td>
</tr>
<tr>
<td>Hypothesis 1</td>
<td>4.33***</td>
</tr>
<tr>
<td>Hypothesis 2</td>
<td>4.82***</td>
</tr>
<tr>
<td>High-yield bond return vs. stock return: $p=4$</td>
<td></td>
</tr>
<tr>
<td>Hypothesis 3</td>
<td>26.38***</td>
</tr>
<tr>
<td>Hypothesis 4</td>
<td>1.98*</td>
</tr>
<tr>
<td>Investment-grade bond return vs. stock return: $p=2$</td>
<td></td>
</tr>
<tr>
<td>Hypothesis 5</td>
<td>17.12***</td>
</tr>
<tr>
<td>Hypothesis 6</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Reported are the $F$ statistics for Granger-causality tests. ***,**, * denote significance at the 1%, 5% and 10% levels, respectively. The lag orders ($p$) are determined by using the BIC information criterion.

Hypothesis 1: High-yield bond returns are not Granger-caused by investment-grade bond returns.
Hypothesis 2: Investment-grade bond returns are not Granger-caused by high-yield bond returns.
Hypothesis 3: High-yield bond returns are not Granger-caused by stock returns.
Hypothesis 4: Stock returns are not Granger-caused by high-yield bond returns.
Hypothesis 5: Investment-grade bond returns are not Granger-caused by stock returns.
Hypothesis 6: Stock returns are not Granger-caused by investment-grade bond returns.
The estimated model is

$$Z_t = [r_{nbbi}, r_{nbbi}]^T$$

where

$$\delta = [\delta_1, \delta_2]^T$$
is the intercept vector,

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix}$$
is the response coefficient matrix,

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} \\ \Psi_{21} & \Psi_{22} & \Psi_{23} \end{bmatrix}$$
is the GARCH-M matrix,

$$H_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix}$$
is conditional variance-covariance vector, $$\epsilon_i | I_{t-1} \sim N(0, H_t)$$, and the conditional variance-covariance matrix

$$H_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix}$$
is assumed to have a BEKK representation, i.e.,

$$H_t = c^T c + a^T \epsilon_{t-1} \epsilon_{t-1}^T + a + g^T H_{t-1} g$$,

where

$$c = \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{21} \end{bmatrix}, a = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, g = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}, \epsilon_i = \begin{bmatrix} \epsilon_{1i} \\ \epsilon_{2i} \end{bmatrix}.$$ The t-statistics are included in parentheses. ***, **, * denote significance at the 1%, 5% and 10% levels, respectively. The intercept vector $$\delta$$ and GARCH-M matrix $$\Psi$$ are transposed in the report for convenience.
Table 9
Nonlinear Granger Causality Tests between Corporate Bond and Stock Excess Returns with Conditional Heteroskedasticity

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Granger-causality test</th>
</tr>
</thead>
<tbody>
<tr>
<td>High-yield bond excess return vs. investment-grade bond excess return: ( p=2 )</td>
<td></td>
</tr>
<tr>
<td>Hypothesis 1</td>
<td>4.36**</td>
</tr>
<tr>
<td>Hypothesis 2</td>
<td>4.83***</td>
</tr>
<tr>
<td>High-yield bond excess return vs. stock excess return: ( p=4 )</td>
<td></td>
</tr>
<tr>
<td>Hypothesis 3</td>
<td>26.34***</td>
</tr>
<tr>
<td>Hypothesis 4</td>
<td>2.01*</td>
</tr>
<tr>
<td>Investment-grade bond excess return vs. stock excess return: ( p=2 )</td>
<td></td>
</tr>
<tr>
<td>Hypothesis 5</td>
<td>16.82***</td>
</tr>
<tr>
<td>Hypothesis 6</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Reported in the table are the \( F \) statistics for Granger-causality tests. ***, **, * denote significance at the 1%, 5% and 10% levels, respectively. The lag orders (\( p \)) are selected by using the BIC information criterion.

Hypothesis 1: High-yield bond excess returns are not Granger-caused by investment-grade bond excess returns.
Hypothesis 2: Investment-grade bond excess returns are not Granger-caused by high-yield bond excess returns.
Hypothesis 3: High-yield bond excess returns are not Granger-caused by stock excess returns.
Hypothesis 4: Stock excess returns are not Granger-caused by high-yield bond excess returns.
Hypothesis 5: Investment-grade bond excess returns are not Granger-caused by stock excess returns.
Hypothesis 6: Stock excess returns are not Granger-caused by investment-grade bond excess returns.
Table 10
Nonlinear Granger Causality Test between Corporate Bond Markets and S&P 500 Futures Markets with Conditional Heteroskedasticity

<table>
<thead>
<tr>
<th>Panel A: Causality tests using returns</th>
<th>Granger-Causality Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hypothesis</td>
<td></td>
</tr>
<tr>
<td>High-yield bond return vs. futures return: $p=3$</td>
<td></td>
</tr>
<tr>
<td>Hypothesis 1</td>
<td>30.33 ***</td>
</tr>
<tr>
<td>Hypothesis 2</td>
<td>2.10 *</td>
</tr>
<tr>
<td>Investment-grade bond return vs. futures return: $p=2$</td>
<td></td>
</tr>
<tr>
<td>Hypothesis 3</td>
<td>15.95 ***</td>
</tr>
<tr>
<td>Hypothesis 4</td>
<td>0.21</td>
</tr>
<tr>
<td>Panel B: Causality tests using excess returns</td>
<td></td>
</tr>
<tr>
<td>High-yield bond excess return vs. futures excess return: $p=3$</td>
<td></td>
</tr>
<tr>
<td>Hypothesis 5</td>
<td>30.23 ***</td>
</tr>
<tr>
<td>Hypothesis 6</td>
<td>1.52</td>
</tr>
<tr>
<td>Investment-grade bond excess return vs. futures excess return: $p=2$</td>
<td></td>
</tr>
<tr>
<td>Hypothesis 7</td>
<td>15.68</td>
</tr>
<tr>
<td>Hypothesis 8</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Reported in the table are $F$ statistics for Granger-causality tests. ***,**,* denote significance at the 1%, 5% and 10% levels, respectively. The lag orders ($p$) in VAR models are selected using the BIC information criterion.

Hypothesis 1: High-yield bond returns are not Granger-caused by futures returns.
Hypothesis 2: Futures returns are not Granger caused by high-yield bond returns.
Hypothesis 3: Investment-grade bond returns are not Granger-caused by high-yield bond returns.
Hypothesis 4: Futures returns are not Granger-caused by investment-grade bond returns.
Hypothesis 5: High-yield bond excess returns are not Granger-caused by futures excess returns.
Hypothesis 6: Futures excess returns are not Granger caused by high-yield bond excess returns.
Hypothesis 7: Investment-grade bond excess returns are not Granger-caused by futures excess returns.
Hypothesis 8: Futures excess returns are not Granger-caused by investment-grade bond excess returns.
### Table 11
Relations between Corporate Bond Returns and Government Bond and Stock Index Returns

<table>
<thead>
<tr>
<th></th>
<th>Test1: $\beta^S_j = 0, \forall j$</th>
<th>Test2: $\beta^S_j = 0, j &gt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{nbbh_t}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 1</td>
<td>$0.353^{***}$</td>
<td>$0.214^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>Model 2</td>
<td>$0.369^{***}$</td>
<td>$0.203^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.020)</td>
</tr>
<tr>
<td></td>
<td>$0.214^{***}$</td>
<td>$0.084$</td>
</tr>
<tr>
<td></td>
<td>(0.074)</td>
<td>(0.098)</td>
</tr>
<tr>
<td></td>
<td>$0.317^{***}$</td>
<td>$76.06^{***}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$43.04^{***}$</td>
</tr>
<tr>
<td>$r_{nbbi_t}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 1</td>
<td>$0.105^{**}$</td>
<td>$0.983^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Model 2</td>
<td>$-0.040$</td>
<td>$1.130^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.052)</td>
<td>(0.061)</td>
</tr>
<tr>
<td></td>
<td>$0.060$</td>
<td>$0.067^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.016)</td>
</tr>
<tr>
<td></td>
<td>$0.285^{***}$</td>
<td>$73.22^{***}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$35.18^{***}$</td>
</tr>
<tr>
<td>$r_{nbbh_t}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 1</td>
<td>$0.373^{***}$</td>
<td>$0.529^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.041)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>Model 2</td>
<td>$0.280^{***}$</td>
<td>$0.122^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.046)</td>
<td>(0.111)</td>
</tr>
<tr>
<td></td>
<td>$0.199^{***}$</td>
<td>$64.24^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.088)</td>
</tr>
<tr>
<td></td>
<td>$1.084^{***}$</td>
<td>$34.82^{***}$</td>
</tr>
<tr>
<td>$r_{nbbi_t}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 1</td>
<td>$0.057$</td>
<td>$1.061^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.049)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>Model 2</td>
<td>$-0.005$</td>
<td>$1.052^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.058)</td>
<td>(0.068)</td>
</tr>
<tr>
<td></td>
<td>$0.061$</td>
<td>$0.063^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.018)</td>
</tr>
<tr>
<td></td>
<td>$28.92^{***}$</td>
<td>$19.12^{***}$</td>
</tr>
<tr>
<td>$r_{nbbh_t}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 1</td>
<td>$0.380^{***}$</td>
<td>$0.162^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.027)</td>
</tr>
<tr>
<td>Model 2</td>
<td>$0.343^{***}$</td>
<td>$0.108^{**}$</td>
</tr>
<tr>
<td>(q4=3)</td>
<td>(0.038)</td>
<td>(0.027)</td>
</tr>
<tr>
<td></td>
<td>$0.200^{***}$</td>
<td>$0.045$</td>
</tr>
<tr>
<td>(q4=3)</td>
<td>(0.022)</td>
<td>(0.013)</td>
</tr>
<tr>
<td></td>
<td>$0.168^{***}$</td>
<td>$0.050^{***}$</td>
</tr>
<tr>
<td>(q4=3)</td>
<td>(0.023)</td>
<td>(0.012)</td>
</tr>
<tr>
<td></td>
<td>$23.74^{***}$</td>
<td>$26.15^{***}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$9.76^{***}$</td>
</tr>
</tbody>
</table>

The table reports the regression results of corporate bond market return on stock market return and intermediate government bond returns. The co-movement between the high-yield bond market and investment-grade bond markets is excluded in Model 1 but considered in Model 2. Standard errors are reported in parentheses. In Test 1, we test whether $\beta^S_j = 0$ for all $j$ including the contemporary return whereas in Test 2 we test whether $\beta^S_j = 0$ for $j > 0$. *, **, *** denotes the significance level at the 10%, 5% and 1%, respectively. The heteroskedasticity in the error term is captured by the GARCH (1,1) process.
Figure 1. Daily Returns and Histograms

A. High-yield bond index

B. Investment-grade bond index

C. S&P 500 stock index

D. S&P 500 index futures